Name:

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Instructions:

- **SECTION ONE: Calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of SECTION ONE: 30 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Question 1 **(8 marks)**

Determine the following integrals:

a) $\int \sin^3(x)\cos^3(x)\, dx$ (4 marks)

b) $\int_0^{\frac{1}{2}} \dfrac{\cos(\pi x)}{2 + \sin(\pi x)}\, dx$ (4 marks)

Question 2 **(6 marks)**

Using the substitution $u = \tan x$ and the identity $\sec^2 x = 1 + \tan^2 x$ determine the following definite integral:

$$\int_{\pi/4}^{\pi/3} \tan^2 x + \tan^4 x \; dx$$

Question 3 (6 marks)

a) Express $\dfrac{2x^2 - 9x + 12}{(x-2)(x-3)}$ in the form $A + \dfrac{B}{x-2} + \dfrac{C}{x-3}$ (3 marks)
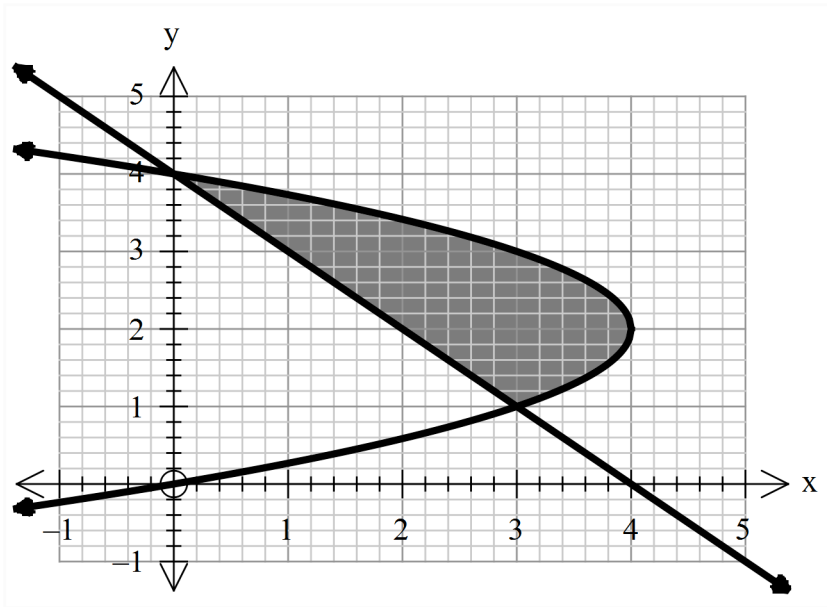
b) Hence determine $\displaystyle\int_{4}^{5} \dfrac{2x^2 - 9x + 12}{(x-2)(x-3)}\, dx$ (3 marks)

Question 4 (6 marks)

The graphs defined by $(y-2)^2 = 4 - x$ *and* $x + y = 4$ are shown below. Calculate the **exact** area enclosed between the two curves as shaded in the diagram below.

Hale School

Mathematics Specialist

Term 3 2018

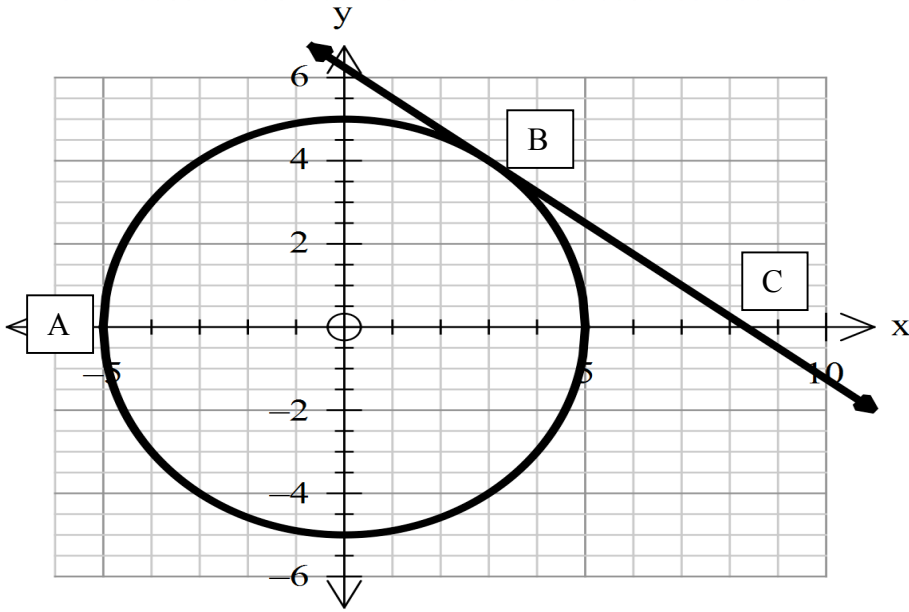### Test 4 - Integration

SECTION TWO

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Name:

Instructions:

- **SECTION TWO: CAS and other calculators are allowed**
- **External notes are not allowed**
- **Duration of SECTION TWO: 15 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

Question 5 (5 marks)

The graph above shows the circle $x^2 + y^2 = 25$ and the line $3x + 4y = 25$ which is a tangent to the circle, touching at point B. Points A and C are x – intercepts for the circle and the line respectively.

The region bounded by the minor arc AB, line segment BC and the x – axis is rotated 360° about the x – axis.

Determine the volume of the resulting solid accurate to 0.1 cubic units.

Question 6 **(7 marks)**
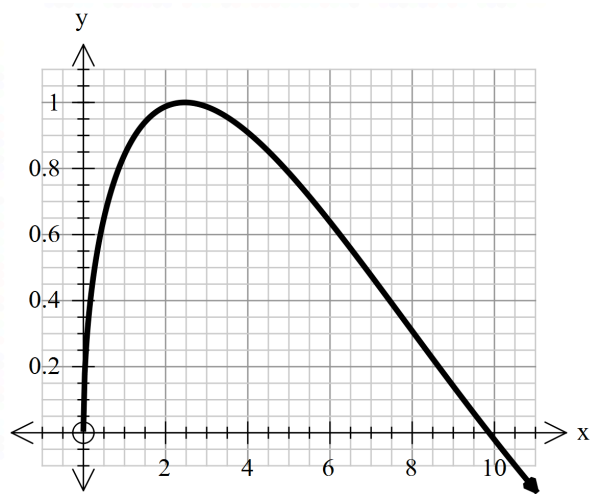
The diagram opposite shows the graph of the function $y = \sin(\sqrt{x})$.

A is the area of the region between the curve and the x – axis.

a) Write down an integral for the value of A and calculate this value to 5 decimal places. (2 marks)

b) Estimate the value of A using 6 midpoint rectangles and state the percentage error for this result accurate to 0.1%. (2 marks)

c) Investigate the number of strips required using midpoint rectangles so that the percentage error between the estimated value and the true result is less than 1%. Show evidence for your answer. (3 marks)

SPARE PAGE FOR EXTRA WORKING